Deliverable 11: Lecture Presentations

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ABSTRACT

This Deliverable 11 of the ETHOS project, is part of WP3 “Dissemination and communication activities” and contains the lecturing presentations of the ETHOS project to the graduate and post-graduate students in the School of Naval Architecture and Marine Engineering and School of Civil Engineering, NTUA.

1. Introduction

Lecture presentations detailing the scientific outcomes of the ETHOS project have been prepared for graduate students of the School of Naval Architecture and Marine Engineering, NTUA, as well as postgraduate students from the School of Naval Architecture and Marine Engineering and the School of Civil Engineering, NTUA. These lectures are delivered by three research members—namely, the Coordinator, Assistant Professor D. Konispoliatis; Professor I. Chatjigeorgiou; and Professor G. Grigoropoulos. The lecture series also incorporates relevant videos showcasing scaled-down constructions and experimental tests conducted in NTUA’s wave tank, thereby elucidating the scientific principles underpinning the proposed project.

The course aims to equip students with:

- The essential theoretical background to comprehend the fundamental phenomena governing the interaction between ocean wave-generated forces and floating or fixed bodies and structures. This includes understanding load generation mechanisms on marine structures based on their size (hydrodynamically “slender” versus volumetric bodies).
- Specialized knowledge in the mathematical modeling of related diffraction and radiation problems, including the presentation and application of fundamental solution methods and techniques.
- Expertise in determining environmental loads on structures through flow dynamics theory.
- The ability to calculate loads on oscillating water column devices with multiple chambers and to evaluate their wave power efficiency.

Upon successful completion of the course, students will be able to:

- Comprehend the critical issues involved in modeling environmental conditions.
- Utilize fundamental results from potential flow theory to determine environmental loads acting on floating structures.
- Apply the numerical methods covered in the course to perform basic calculations of wave fields, applied loads on floating structures—particularly Oscillating Water Column devices—and their responses due to interactions with ocean waves.

Laboratory for Floating Structures and Mooring Systems,
School of Naval Architecture and Marine Engineering,
National Technical University of Athens (NTUA)

ETHOS: Novel Type Offshore Floating Wave Energy Converter for Efficient Power Absorption

Asst. Prof. D.N. Konispoliatis

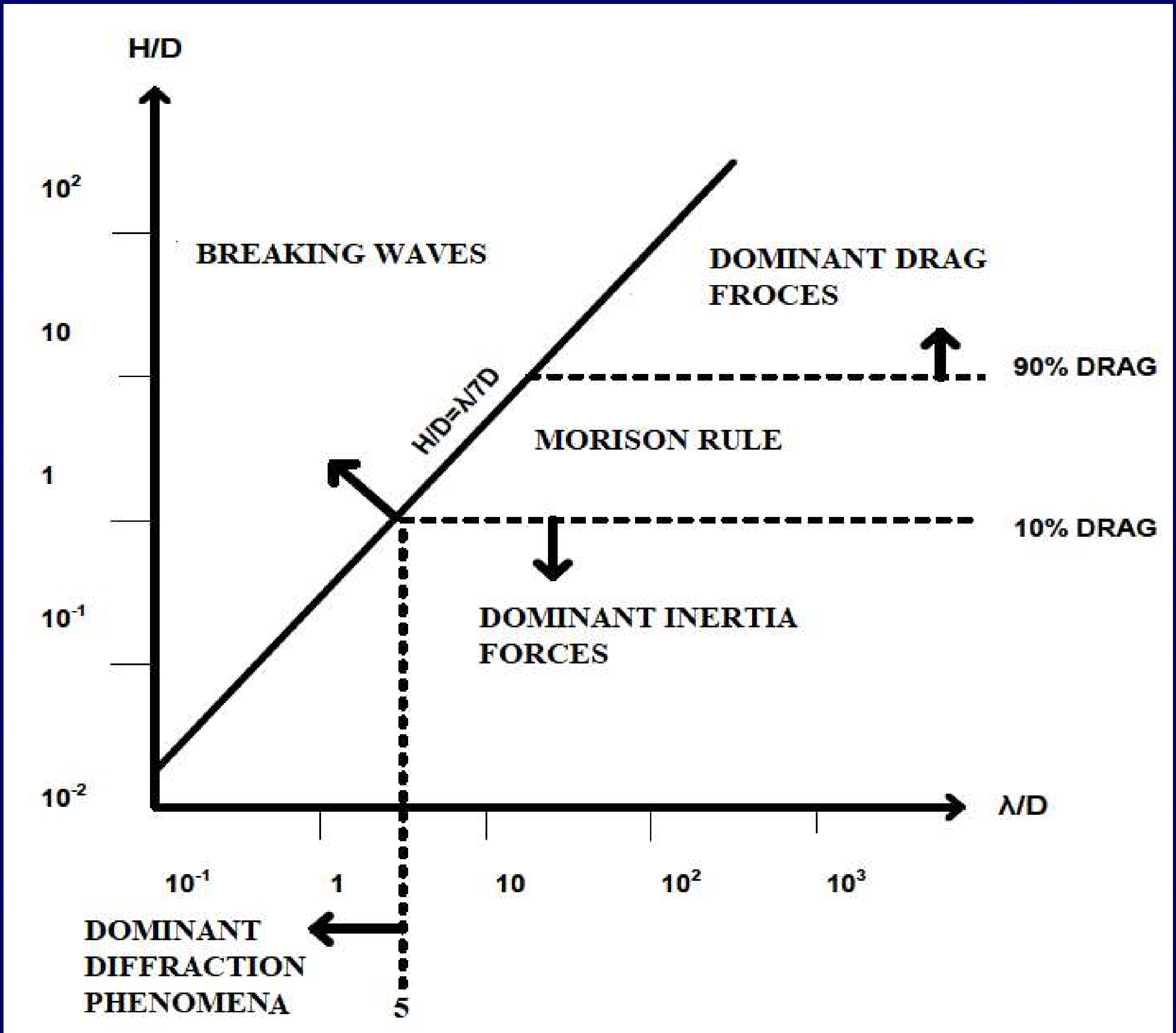
Ethos

H.F.R.I. Hellenic Foundation for Research & Innovation

Greece 2.0 NATIONAL RECOVERY AND RESILIENCE PLAN

Funded by the European Union NextGenerationEU

SEA LOADS ON MARINE STRUCTURES



The body is assumed "large" if λ/D<5

For H/D>10 the acting loads on the solid consist of 90% of drag forces and 10% inertia forces

The limit H/D=λ/(7D) is the Michell – Havelock limit of breaking waves in deep water

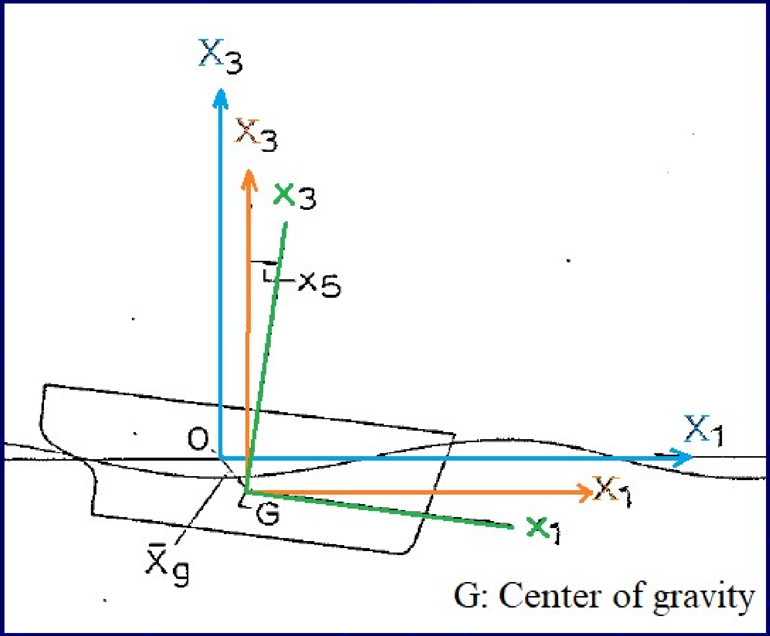
POTENTIAL FLOW THEORY

"*When a flow is both inviscid and irrotational, pleasant things happen*" – F. M. White, Fluid Mechanics 4th ed.

We can treat external flows around bodies as inviscid (i.e. frictionless), irrotational (i.e. the fluid particles are not rotating) and incompressible. Thus, we can define a potential function *φ(x,z,t)*, as a continuous function that satisfies the basic laws of fluid mechanics, and equals to:

$$\Phi(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3; t) = \Phi_0(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3; t) + \Phi_s(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3; t)$$

Where Φ_0 is the undisturbed velocity potential of the incoming wave and Φ_s is the scattered velocity potential due to the presence of the body.



The origin of the $\mathrm{G}-\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3$ coordinate system is on the CG.
The origin of the $\mathrm{O}-\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$ coordinate system is on the undisturbed water surface level
The origin of the $\mathrm{G}-\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$ coordinate system is on the CG and the axes are always parallel to $\mathrm{O}-\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$

POTENTIAL FLOW THEORY

As being also known, the velocity potential must satisfy the Laplace equation:

$$\Delta\Phi = 0 \leftrightarrow \nabla^2\Phi = 0 \leftrightarrow \left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}\right) = 0$$

as well as the below boundary conditions:

- on the sea bottom: $\frac{\partial\Phi}{\partial\vec{n}} = 0;\ \mathrm{X}_3 = -d$
- on the body's wetted surface: $\frac{\partial\Phi}{\partial\vec{N}} = V_N;$ V_N is the fluid's velocity at the *N* direction; *N* is the vertical to the solid's surface vector expressed on the $\mathrm{O}-\mathrm{X}_1,\mathrm{X}_2,\mathrm{X}_3$ coordinate system
- Kinematic and dynamic boundary conditions:

$$\frac{\partial\zeta}{\partial t} + \frac{\partial\Phi}{\partial X_1}\frac{\partial\zeta}{\partial X_1} + \frac{\partial\zeta}{\partial X_2}\frac{\partial\Phi}{\partial X_2} - \frac{\partial\Phi}{\partial X_3} = 0;\ \text{for } X_3 = \zeta(X_1, X_2; t)$$

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial X_1}\right)^2 + \left(\frac{\partial\Phi}{\partial X_2}\right)^2 + \left(\frac{\partial\Phi}{\partial X_3}\right)^2\right] + gX_3 = 0;\ \text{for } X_3 = \zeta(X_1, X_2; t)$$

- at infinity, stating that propagating disturbances must be outgoing

POTENTIAL FLOW THEORY

The linearization procedure of the kinematic and dynamic boundary conditions is well described in the corresponding course (refer to the “Analysis and Design of Floating Structures”). Thus, it is no further elaborated here.

Then we linearize the boundary condition on the body’s wetted surface.

But, before that we will present the kinematics of a fluid particle on the body’s wetted surface

Click to continue

POTENTIAL FLOW THEORY

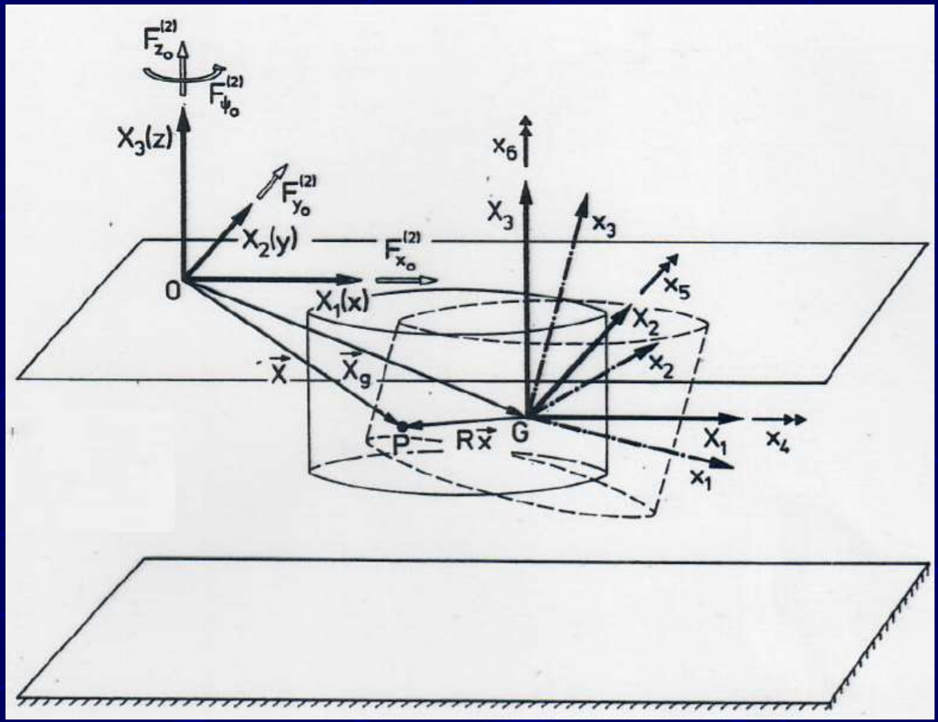
Assuming that a fluid particle P is placed on the body's wetted surface, its position vector relevant to the undisturbed co-ordinate system $\mathrm{O} - \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$ can be written as:

$$\vec{X} = \vec{X}^{(0)} + \varepsilon \vec{X}^{(1)} + \varepsilon^2 \vec{X}^{(2)}$$

Herein, small motions of the body around its mean position is assumed.
The term $\varepsilon \vec{X}^{(1)}$ denotes the first order displacement of the particle P from its mean position $\vec{X}^{(0)}$ due to the first order acting wave loads on the body. Similar for the $\varepsilon^2 \vec{X}^{(2)}$ term.

Each of the $\vec{X}^{(0)}, \vec{X}^{(1)}, \vec{X}^{(2)}$ motions can be written decomposed into two terms: a) the translational motions, X_{g1}, X_{g2}, X_{g3} of the CG on the $\mathrm{O} - \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$ co ordinate system; and b) the rotational motions around the coordinate system $\mathrm{G} - \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$, according to the following relation:

$$\vec{X}^{(j)} = \vec{X}_g^{(j)} + R^{(j)} \vec{x}, \; j\text{=0,1,2}$$



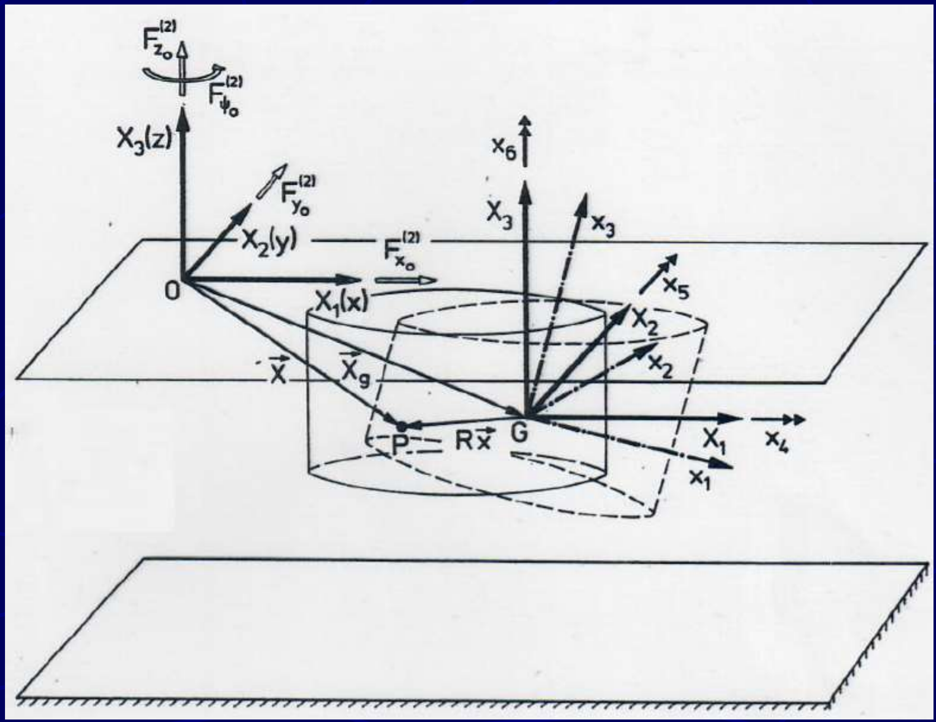
POTENTIAL FLOW THEORY

$$\vec{X}^{(j)} = \vec{X}_g^{(j)} + R^{(j)}\vec{x},\ j{=}0,1,2$$

Herein, $\vec{X}_g^{(j)}$ denotes the translational motions of the CG; $R^{(j)}$ is the rotational matrix, containing the rotation angles; $\vec{x}$ is the position vector of the P point relevant to $\mathrm{G}-\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$.

It holds: $R^{(j)} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{31} & a_{33} \end{bmatrix}$ whereas:

$$\begin{cases} a_{11} = \cos x_5 \cos x_6 \\ a_{12} = \sin x_4 \sin x_5 \cos x_6 - \cos x_4 \sin x_6 \\ a_{13} = cos x_4 \sin x_5 \cos x_6 + \sin x_4 \sin x_6 \\ a_{21} = \cos x_5 \sin x_6 \\ a_{22} = \sin x_4 \sin x_5 \sin x_6 + \cos x_4 \cos x_6 \\ a_{23} = \cos x_4 \sin x_5 \sin x_6 - \sin x_4 \cos x_6 \\ a_{31} = -\sin x_5 \\ a_{32} = \sin x_4 \cos x_5 \qquad a_{33} = \cos x_4 \cos x_5 \end{cases}$$



POTENTIAL FLOW THEORY

Applying Taylor series on the trigonometric functions, the rotation matrix R can be written, for the zero –order (calm position); first- and second- order terms as:

$$R^{(0)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad R^{(1)} = \begin{bmatrix} 0 & -x_6^{(1)} & x_5^{(1)} \\ x_6^{(1)} & 1 & -x_4^{(1)} \\ -x_5^{(1)} & x_4^{(1)} & 1 \end{bmatrix}$$

$$R^{(2)} = \begin{bmatrix} -\frac{1}{2}\left[(x_5^{(1)})^2+(x_6^{(1)})^2\right] & -x_6^{(2)} + x_4^{(1)}x_5^{(1)} & x_5^{(2)} + x_4^{(1)}x_6^{(1)} \\ x_6^{(1)} & -\frac{1}{2}\left[(x_4^{(1)})^2+(x_6^{(1)})^2\right] & -x_4^{(1)} + x_5^{(1)}x_6^{(1)} \\ -x_5^{(1)} & -x_4^{(1)} & -\frac{1}{2}\left[(x_4^{(1)})^2+(x_5^{(1)})^2\right] \end{bmatrix}$$

POTENTIAL FLOW THEORY

The zero order term: $\vec{X}^{(0)} = \vec{X}_g^{(0)} + R^{(0)}\vec{x} = \begin{bmatrix} X_{g1}^{(0)} \\ X_{g2}^{(0)} \\ X_{g3}^{(0)} \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ denotes the coordinates of the P point, at its calm position, relevant to the $\mathrm{O} - \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3$ coordinate system

The first order term:

$$\vec{X}^{(1)} = \vec{X}_g^{(1)} + R^{(1)}\vec{x} = \begin{bmatrix} X_{g1}^{(1)} \\ X_{g2}^{(1)} \\ X_{g3}^{(1)} \end{bmatrix} + \begin{bmatrix} -x_6^{(1)}x_2 + x_5^{(1)}x_3 \\ x_6^{(1)}x_1 - x_4^{(1)}x_3 \\ -x_5^{(1)}x_1 + x_4^{(1)}x_2 \end{bmatrix}$$

Similar for the second order term …

POTENTIAL FLOW THEORY

The linearization procedure of the kinematic and dynamic boundary conditions is well described in the corresponding course (refer to the “Analysis and Design of Floating Structures”). Thus, it is no further elaborated here.

Then we linearize the boundary condition on the body’s wetted surface.

But, before that we will present the kinematics of a fluid particle on the body’s wetted surface

The boundary condition on the body’s wetted surface denotes that the relevant velocity of the fluid and the body’s wetted surface equals to zero on the direction of the vertical to the body’s surface vector V. This physically means that the fluid particles do not enter inside the solid.

Thus, the equation : $\frac{\partial \Phi}{\partial \vec{N}} = V_N$; can be written as: $\overrightarrow{\nabla\Phi}\vec{N} = \vec{V}\vec{N}$

The fluid velocity on the body’s surface, can be written as: $\vec{V} = \dot{\vec{X}} = \varepsilon\dot{\vec{X}}^{(1)} + \varepsilon^2\dot{\vec{X}}^{(2)}$ (1)

The zero-order terms does not appear in the equation, since $\vec{X}^{(0)}$ is independent from the time (t).

POTENTIAL FLOW THEORY

After keeping the first-order terms, it holds:

$$\vec{V} = \varepsilon \dot{\vec{X}}^{(1)}; \text{ here } \vec{X}^{(1)} = \vec{X}_g^{(1)} + R^{(1)}\vec{x}$$

Similar for the vertical vector *N*: $\vec{N} = \vec{N}^{(0)} + \varepsilon \vec{N}^{(1)} + \varepsilon^2 \vec{N}^{(2)}$ (2)

Furthermore, it is well known that: $\Phi(x, y, z; t) = \sum_{n=1}^{\infty} \varepsilon^n \Phi^{(n)}(x, y, z; t)$, where, holding the first order terms, we obtain: $\Phi = \Phi_0^{(1)} + \Phi_s^{(1)}$

The $\Phi_s^{(1)}$ term equals to the sum of the diffraction and the radiation potential, i.e.

$$\Phi_s^{(1)} = \Phi_7^{(1)} + \Phi_R^{(1)} \qquad (3)$$

Diffracted flow due to the presence of the body (body is assumed restrained in the wave impact)

Radiated flow due to the motions of the body in the wave impact

POTENTIAL FLOW THEORY

Thus, the equation: $\overrightarrow{\nabla\Phi}\vec{N} = \vec{V}\vec{N}$ can be written, for the first-order terms, as:

$$\left(\vec{\nabla}\Phi_0^{(1)} + \vec{\nabla}\Phi_7^{(1)} + \vec{\nabla}\Phi_R^{(1)}\right)\vec{n} = \overrightarrow{V^{(1)}}\vec{n}$$

or by decomposing:

$$\vec{\nabla}\left(\Phi_0^{(1)} + \Phi_7^{(1)}\right)\vec{n} = 0$$

Diffraction boundary condition on the body's wetted surface

$$\vec{\nabla}\left(\Phi_R^{(1)}\right)\vec{n} = \overrightarrow{V^{(1)}}\vec{n}$$

Radiation boundary condition on the body's wetted surface

POTENTIAL FLOW THEORY

We will further analyze the term $\overrightarrow{V^{(1)}}\vec{n}$, i.e.

$$\overrightarrow{V^{(1)}}\vec{n} = V_1^{(1)} n_1 + V_2^{(1)} n_2 + V_3^{(1)} n_3 = \left(\dot{X}_{g1}^{(1)} - \dot{x}_6 x_2 + \dot{x}_5 x_3\right) n_1 +$$

$$+\left(\dot{X}_{g2}^{(1)} + \dot{x}_6 x_1 - \dot{x}_4 x_3\right) n_2 + \left(\dot{X}_{g3}^{(1)} + \dot{x}_4 x_2 - \dot{x}_5 x_1\right) n_3 = \dot{X}_{g1}^{(1)} n_1 + \dot{X}_{g2}^{(1)} n_2 + \dot{X}_{g3}^{(1)} n_3 +$$

$$+(x_2 n_3 - x_3 n_2)\dot{x}_4 + (x_3 n_1 - x_1 n_3)\dot{x}_5 + ($$

$$\dot{X}_g^{(1)} + \dot{R}\vec{x} = \begin{bmatrix} \dot{X}_{g1}^{(1)} \\ \dot{X}_{g2}^{(1)} \\ \dot{X}_{g3}^{(1)} \end{bmatrix} + \begin{bmatrix} 0 & -\dot{x}_6 & \dot{x}_5 \\ \dot{x}_6 & 0 & -\dot{x}_4 \\ -\dot{x}_5 & \dot{x}_4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Thus,

$$\overrightarrow{V^{(1)}} \quad j=1$$

where $(n_1, n_2, n_3) = \vec{n}$ and $(n_4, n_5, n_6) = \vec{r}\ x\ \vec{n}$

and $\dot{x}_j = (\dot{x}_{g1}, \dot{x}_{g2}, \dot{x}_{g3})$ for *j=1,2,3* and $\dot{x}_j = \left(\dot{x}_{g4}, \dot{x}_{g5}, \dot{x}_{g6}\right)$ for *j=4,5,6*

Here, $\vec{r}$ is the position vector with respect to the origin of the coordinate system.

In the framework of linearity, we assume that:

$$\Phi_R^{(1)} = \sum_{j=1}^{6} \dot{x}_j\, \Phi_j^{(1)}$$

POTENTIAL FLOW THEORY

The velocity potentials and the body's motions can be also written as:

$$\Phi_j^{(1)} = \mathrm{Re}\left\{\varphi_j^{(1)}\right\} e^{-i\omega t}; j = 0,1, \dots, 7 \qquad x_j^{(1)} = \mathrm{Re}\left\{x_{j0}^{(1)}\right\} e^{-i\omega t}; j = 1, \dots, 6$$

The first-order potential around a freely floating cylindrical body, can be written as:

$$\varphi = \varphi_0^{(1)} + \varphi_7^{(1)} + \sum_{j=1}^{6} \dot{x}_{j0}\, \varphi_j^{(1)}$$

Laplace: $\Delta\varphi = 0 \leftrightarrow \nabla^2\varphi = 0 \leftrightarrow \left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}\right) = 0$

Sea bottom: $\frac{\partial\varphi}{\partial z} = 0, z = -d$

Free surface: $\omega^2\varphi - g\frac{\partial\varphi}{\partial z} = 0, \quad z = 0$

Body's wetted surface: $\frac{\partial\varphi_7^{(1)}}{\partial\vec{n}} = -\frac{\partial\varphi_0^{(1)}}{\partial\vec{n}}, \qquad \frac{\partial\varphi_j^{(1)}}{\partial\vec{n}} = n_j$

Boundary condition at infinity

The total potential, φ, describing the fluid flow around a structure in the presence of waves, equals the sum of the undisturbed wave potential, φ_o, the potential due to the presence of the body when it is motionless, φ_7, and the radiation potential, φ_j caused by the body motion.

POTENTIAL FLOW THEORY

One of the primary reasons for studying the fluid motion past a body is our desire to predict the forces and moments acting on the body due to the dynamic pressure of the fluid. Thus, we wish to consider the six components of the force and moment vectors, which are represented by the integrals of the pressure over the body surface, or

$$\mathrm{F} = \iint_S \mathrm{Pn}dS \qquad \mathrm{M} = \iint_S \mathrm{P(rxn)}dS$$

Here, the normal vector n is taken to be positive when pointing out of the fluid volume and hence into the body; S is the body's wetted surface.
The pressure derives from the Bernoulli equation, i.e.

The fluid pressure on a random location on the body's wetted surface can be derived by:

$$\mathrm{P}(X_1, X_2, X_3; \mathrm{t}) = -\rho g X_3 - \rho \Phi_t - \frac{1}{2}\rho\left|\vec{\nabla}\Phi\right|^2$$

Assuming that the

$$\overrightarrow{F^{(1)}} = -\iint_S \rho g X_3 \vec{n} dS - \iint_S \rho \Phi_t \vec{n} dS$$

FIRST-ORDER FORCES/MOMENTS

Similarly, also the first order moments can be derived. Thus, the first order forces and moments can be written as:

$$F_k = -\iint_S \rho g X_3 n_k dS - \iint_S \rho \Phi_t n_k dS\,, k = 1,2,3 \text{ and } k = 4,5,6$$

Substituting the velocity potential into the above equation, it holds:

$$F_k = -F_{k_{st}} - i\omega\rho e^{-i\omega t} \iint_S \left[\varphi_0 + \varphi_7 + \sum_{j=1}^{6} \dot{x}_{j0}\varphi_j\right] n_k dS\,, k = 1,2,3 \text{ and } k = 4,5,6$$

The first term in the above equation denotes the hydrostatic forces on the body, whereas, the second term of the equation denotes the hydrodynamic forces acting on the body.

HYDROSTATIC FORCES/MOMENTS

The hydrostatic forces and moments equal to:

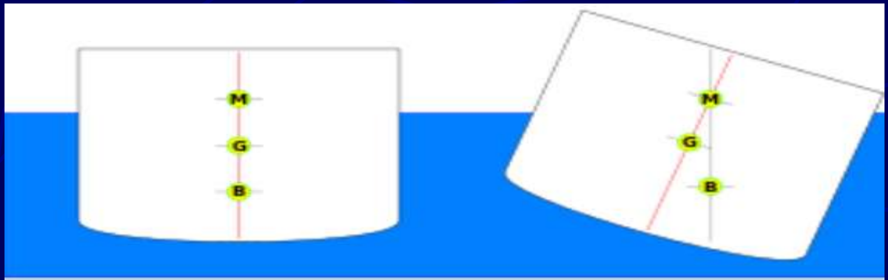
$$F_{k_{st}}(t) = -C_{kj}X_j, \text{for } k, j = 1,2, \dots, 6$$

Here X_j are the body's motions/rotations at its 6-degrees of freedom, whereas, C_{kj} are the restoring coefficients.

Assuming that the examined body has a single plane of symmetry x_1-x_3, i.e. cylindrical bodies, the non zero restoring coefficients are:

$$C_{33} = \rho g A_{WL}; \quad C_{53} = C_{35} = -\rho g \iint_{A_{WL}} X_1 dA_{WL}; \quad C_{44} = \rho g \nabla G\overline{M_T}; \quad C_{55} = \rho g \nabla G\overline{M_L}$$

Here A_{WL} is the water line surface, V is the displace fluid volume, GM_T, GM_L, are the transverse and longitudinal metacentric height.



Ship stability diagram showing *centre of gravity* (G), *centre of buoyancy* (B), and *metacentre* (M) with ship upright and heeled over to one side.
As long as the load of a ship remains stable, G is fixed. For small angles M can also be considered to be fixed, while B moves as the ship heels.

HYDRODYNAMIC FORCES/MOMENTS

The hydrodynamic forces and moments can be divided to:

a) The exciting forces, derived from the diffraction problem, i.e.

$$F_{k,exciting} = -i\omega\rho e^{-i\omega t} \iint_S [\varphi_0 + \varphi_7] n_k dS \,, k = 1{,}2{,}3 \text{ and } k = 4{,}5{,}6$$

b) The hydrodynamic drag forces of the fluid due to the accelerating motion of the body, i.e.

$$F_{k,hydro} = -i\omega\rho e^{-i\omega t} \iint_S \left[\sum_{j=1}^{6} \dot{x}_{j0}\varphi_j\right] n_k dS \,, k = 1{,}2{,}3 \text{ and } k = 4{,}5{,}6$$

By introducing, the:

$$-\rho \iint_S [\varphi_j] n_k dS = a_{kj} + \frac{i}{\omega} b_{kj}$$

we can write that:

$$F_{kj,hydro} = -a_{kj}\ddot{X}_j - b_{kj}\dot{X}_j$$

where, α_{kj} and b_{kj} are the hydrodynamic added mass and damping coefficient , respectively, at the k direction due to the motion of the body in the j-th direction.

The first term of the exciting forces denotes the Froude-Krylov forces, whereas, the second term denotes the scattered/diffracted waves due to the presence of the body. In the Morison equation, the second term is assumed negligible.

HYDRODYNAMIC FORCES/MOMENTS

Thus, 36 (i.e. 6x6) hydrodynamic coefficients can be defined.

$$a_{kj}\,(or\ b_{kj}) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}$$

The hydrodynamic coefficients are symmetric, thus, it holds:

$$a_{kj} = a_{jk} \text{ and } b_{kj} = b_{jk}$$

We introduce the below mass matrix (i.e. 6x6) which for single plane of symmetry x_1-x_3, equals to:

$$m_{kj} = \begin{bmatrix} m & 0 & 0 & 0 & mX_{g3} & 0 \\ 0 & m & 0 & -mX_{g3} & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mX_{g3} & 0 & J_{44} & 0 & -J_{46} \\ mX_{g3} & 0 & 0 & 0 & J_{55} & 0 \\ 0 & 0 & 0 & -J_{64} & 0 & J_{66} \end{bmatrix}$$

where m is the mass of the body; J_{ii} are the moment of inertia; $(0,0,X_{g3})$ are the coordinates of the CG relevant to $0\text{-}X_1,X_2,X_3$

MOTION EQUATION

From the Newton's 2nd law it holds:

$$\sum_{j=1}^{6} m_{kj}\ddot{X}_j = F_{k_{st}} + F_{kj,hydro} + F_{k,exciting}$$

$$\sum_{j=1}^{6} m_{kj}\ddot{X}_j + a_{kj}\ddot{X}_j + b_{kj}\dot{X}_j + C_{kj}X_j = F_{k,exciting}$$

$$\sum_{j=1}^{6} (m_{kj} + a_{kj})\ddot{X}_j + b_{kj}\dot{X}_j + C_{kj}X_j = F_{k,exciting}$$

which for single plane of symmetry x_1-x_3, equals to:

$$(m + a_{33})\ddot{X}_3 + b_{33}\dot{X}_3 + gA_{WL}X_3 = F_{3,exciting}(t)$$

$$(m + a_{11})\ddot{X}_1 + b_{11}\dot{X}_1 + (mX_{g3} + a_{15})\ddot{X}_5 + b_{15}\dot{X}_5 = F_{1,exciting}(t)$$

$$(I_{55} + a_{55})\ddot{X}_5 + b_{55}\dot{X}_5 + (mX_{g3} + a_{51})\ddot{X}_1 + b_{51}\dot{X}_1 + g\nabla G\overline{M_L}X_5 = F_{5,exciting}(t)$$

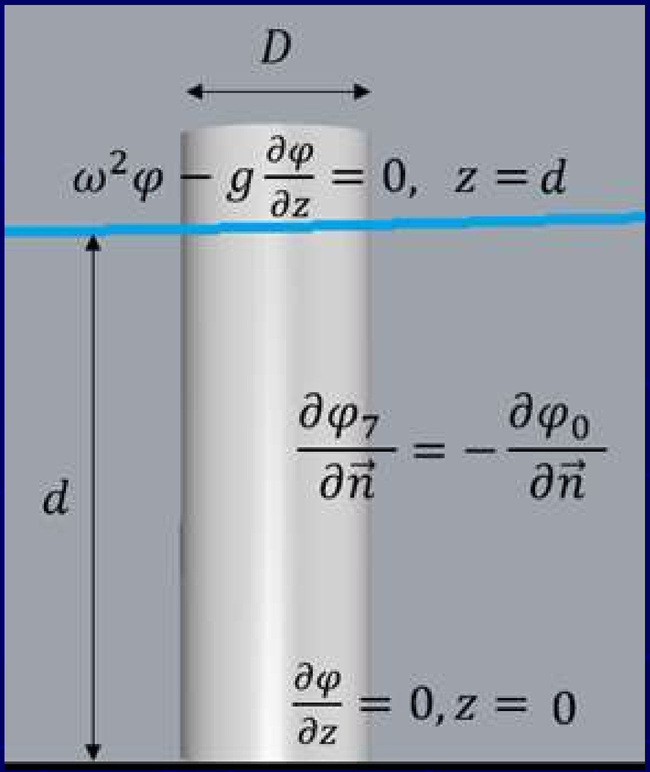
Vertical Bottom Seated Cylinder (McCamy-Fuchs)

It is known that the wave elevation and the velocity potential are derived by:

$$\zeta_0(x,t) = Re\left[\frac{H}{2}e^{i(kx-\omega t)}\right]$$

$$\Phi_0(\mathrm{x},\mathrm{z};\mathrm{t}) = \mathrm{Re}\left[-\frac{iH}{2}\frac{g}{\omega}\frac{\cosh(kz)}{\cosh(kd)}e^{i(kx-\omega t)}\right] = \mathrm{Re}\left[\varphi_0(\mathrm{x},\mathrm{z})e^{-i\omega t}\right]$$

$$\text{where } \varphi_0 = -\frac{iH}{2}\frac{g}{\omega}\frac{\cosh(kz)}{\cosh(kd)}e^{ikx}$$



The coefficient $\varphi = \varphi_0 + \varphi_7$ should form the solution of Laplace equation, i.e.:

$$\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2}\right) = 0$$

and satisfy the boundary conditions on the sea bottom, the free water surface, and a radiation condition at infinity:

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial\varphi_7}{\partial r} - ik\varphi_7\right) = 0$$

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

We introduce cylindrical coordinates: $x = rcos\theta, y = rsin\theta$

Thus,

$$e^{ix} = cosx + isinx \rightarrow \cos(rcos\theta) + isin(rcos\theta)$$

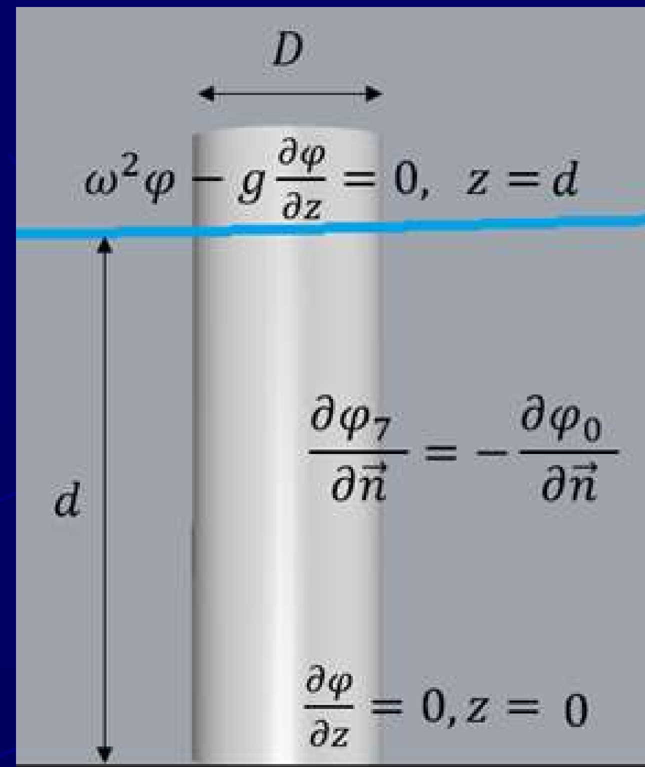
$$e^{iy} = cosy + isiny \rightarrow \cos(rsin\theta) + isin(rsin\theta)$$

Using Bessel series we can write:

$$\cos(rcos\theta) = J_0(r) + 2\sum_{m=1}^{\infty}(-1)^m J_{2m}(r)\cos(2m\theta)$$

$$\sin(\underbrace{rcos\theta}_{x}) = 2\sum_{m=0}^{\infty}(-1)^m J_{2m+1}(r)\cos((2m+1)\theta)$$

$$J_{2m} = \frac{1}{\pi}\int_0^{\pi} \cos(rsin\theta)\cos(2m\theta)\,du \qquad J_{2m+1} = \frac{1}{\pi}\int_0^{\pi} \sin(rsin\theta)\sin((2m+1)\theta)\,du$$



Vertical Bottom Seated Cylinder (McCamy-Fuchs)

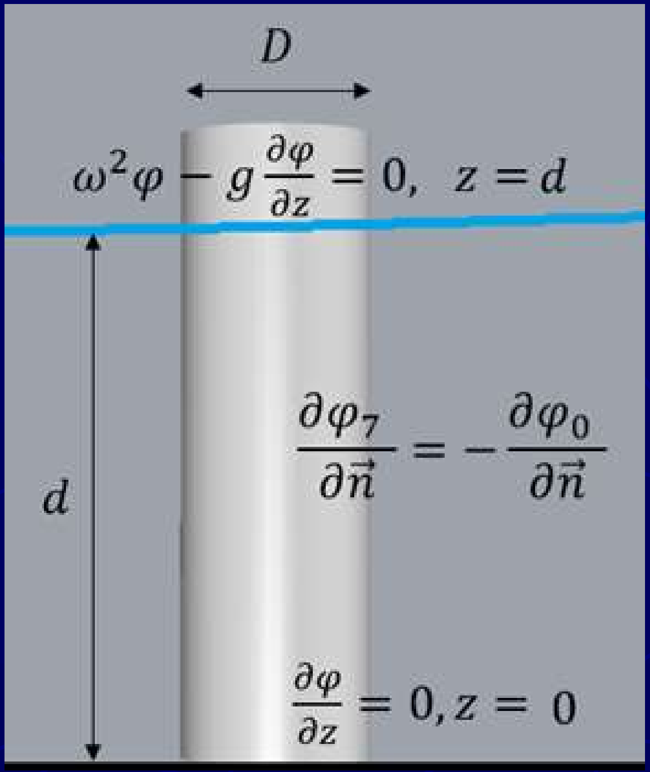
Hence: $e^{ix} = \cos(r\cos\theta) + i\sin(r\cos\theta) =$

$$= J_0(r) + 2\sum_{m=1}^{\infty}(-1)^m J_{2m}(r)\cos(2m\theta) +$$

$$+2i\sum_{m=0}^{\infty}(-1)^m J_{2m+1}(r)\cos((2m+1)\theta) =$$

$$= J_0(r) + 2iJ_1(r)\cos(\theta) + 2i^2J_2(r)\cos(2\theta) + \cdots +$$

$$+2i^m J_m(r)\cos(m\theta)$$



So:

$$\boxed{e^{ikx} = e^{ikr\cos\theta} = \sum_{m=0}^{\infty}\varepsilon_m i^m J_m(kr)\cos(m\theta)}$$

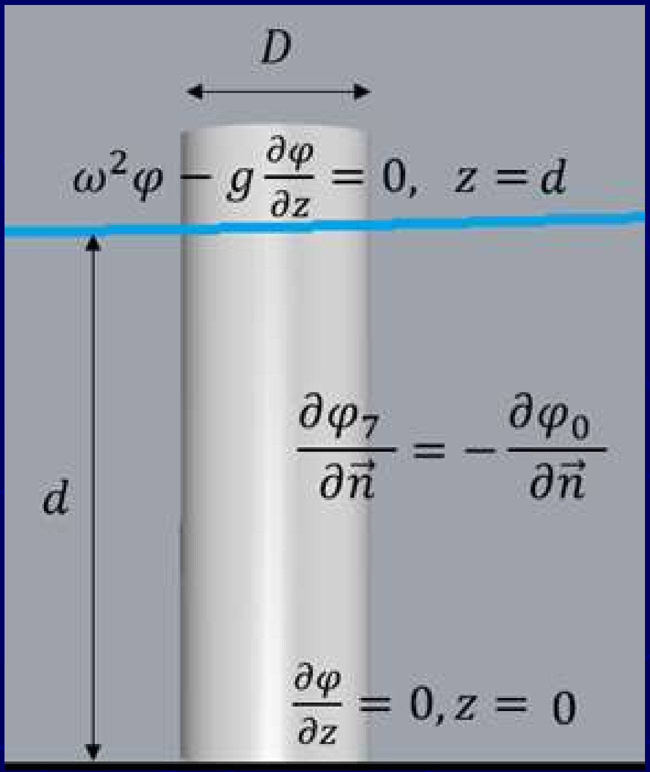
Jacobi – Anger

Here ε_m corresponds to Neumann symbol, $\varepsilon_m = 1, m = 0$ and $\varepsilon_m = 2, m > 0$

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

Thus:

$$\varphi_0(r,\theta,z) = -i\,\frac{gH}{2\omega}\frac{\cosh(kz)}{\cosh(kd)}\sum_{m=0}^{\infty}\varepsilon_m i^m J_m(kr)\cos(m\theta)$$



It holds that: $H_m(r) = J_m(r) + iY_m(r)$

where $\quad Y_m(r) = \dfrac{J_m(r)\cos(m\pi) - J_{-m}(r)}{\sin(m\pi)}$

$$J_{-m}(r) = (-1)^m J_m$$

Here H_m stands for the mth order Hankel function of first kind.

It remains to calculate the potential φ_7. The method of determination is similar to that for the calculation of the undisturbed flow potential carried out in the first lessons.

More specifically, following the method of separable variables:

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

The velocity potential forms: $\varphi_7(r,\theta,z) = \mathrm{R}(r)\cdot\Theta(\theta)\cdot\mathrm{Z}(z)$

After substituting in the Laplace equation we get:

$$\left(\frac{\partial^2\varphi_7}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_7}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi_7}{\partial\theta^2}+\frac{\partial^2\varphi_7}{\partial z^2}\right)=0\rightarrow$$

$$\frac{1}{\mathrm{R}}\left(\frac{\partial^2\mathrm{R}}{\partial r^2}+\frac{1}{r}\frac{\partial\mathrm{R}}{\partial r}\right)+\frac{1}{r^2\Theta}\frac{\partial^2\Theta}{\partial\theta^2}+\frac{1}{\mathrm{Z}}\frac{\partial^2\mathrm{Z}}{\partial z^2}=0 \qquad (5)$$

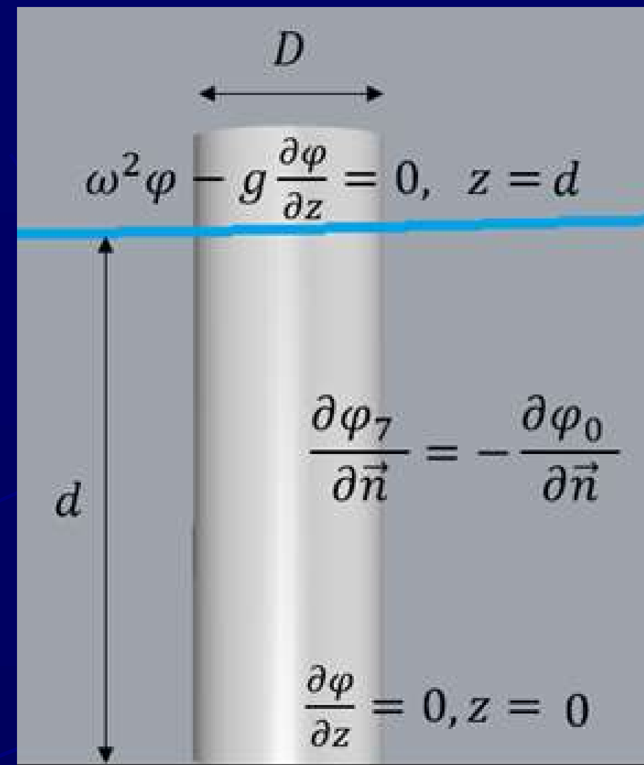
In order equation (5) to be valid:

$$\frac{1}{\Theta}\frac{\partial^2\Theta}{\partial\theta^2}=-m^2\ \text{και}\ \frac{1}{\mathrm{Z}}\frac{\partial^2\mathrm{Z}}{\partial z^2}=k^2, k,m\in R,\text{hence:}$$

$$\frac{\partial^2\Theta}{\partial\theta^2}+\Theta m^2=0\ \text{και}\ \frac{\partial^2\mathrm{Z}}{\partial z^2}-k^2\mathrm{Z}=0 \qquad (6)$$

$$\Theta(\theta)=Asin(m\theta)+Bcos(m\theta)\ \ \text{και}\ Z(z)=Ce^{kz}+De^{-k}$$

It holds $\Theta(\theta)$= $\Theta(-\theta)$ i.e.,: $\Theta(\theta)=Bcos(m\theta)\xrightarrow{B=1}\cos(m\theta)$



Vertical Bottom Seated Cylinder (McCamy-Fuchs)

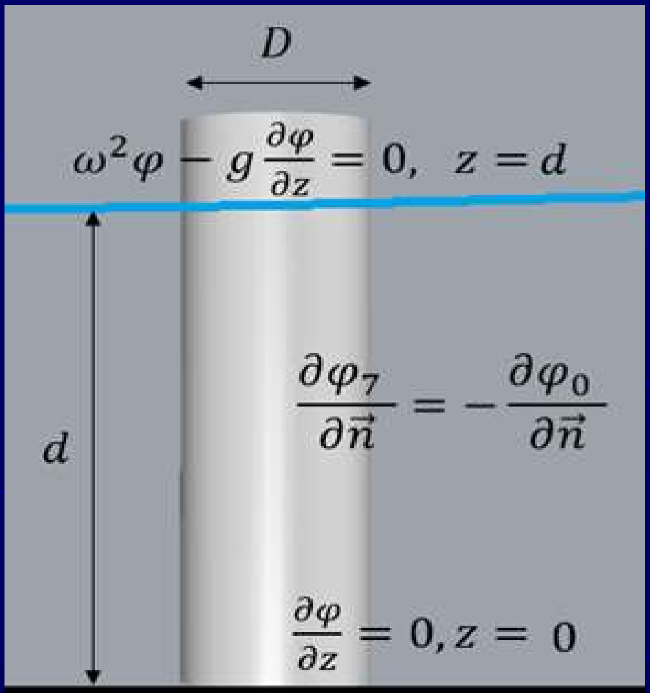
We apply the boundary condition on the seabed:

$$\frac{\partial Z}{\partial z} = 0, Cke^{kz} - Dke^{-kz} = 0, C = D, \text{hence}$$

$$Z(z) = C\left(e^{kz} + e^{-kz}\right) = Ccosh(kz) \overset{C=1}{\Longrightarrow} \cosh(kz)$$

From Equation (5) it is derived:

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + (k^2 - \frac{m^2}{r^2})R = 0 \qquad (6)$$



Eq. (6) has a solution in the form: $R(r) = C_1 J_m(kr) + C_2 Y_m(kr)$

Hence the scattered velocity potential can be written as:

$$\varphi_7(r, \theta, z) = \sum_{m=0}^{\infty}(C_1 J_m(kr) + C_2 Y_m(kr))\cosh(kz)\cos(m\theta)$$

The boundary condition at infinity has not yet been used. To use it, we will calculate the limit values of Bessel functions for large arguments :

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

$$J_m(kr)\big|_{r\to\infty} \approx \sqrt{\frac{2}{\pi kr}}\cos\left(kr - \frac{1}{2}m\pi - \frac{1}{4}\pi\right), \Upsilon_m(kr)\big|_{r\to\infty} \approx \sqrt{\frac{2}{\pi kr}}\sin\left(kr - \frac{1}{2}m\pi - \frac{1}{4}\pi\right)$$

Thus:

$$\dot{J}_m(kr)\big|_{r\to\infty} = -\frac{1}{2}\frac{1}{r}\sqrt{\frac{2}{\pi kr}}\cos\left(kr - \frac{1}{2}m\pi - \frac{1}{4}\pi\right) - k\sqrt{\frac{2}{\pi kr}}\sin\left(kr - \frac{1}{2}m\pi - \frac{1}{4}\pi\right)$$

$$\dot{Y}_m(kr)\big|_{r\to\infty} = -\frac{1}{2}\frac{1}{r}\sqrt{\frac{2}{\pi kr}}\sin\left(kr - \frac{1}{2}m\pi - \frac{1}{4}\pi\right) + k\sqrt{\frac{2}{\pi kr}}\cos\left(kr - \frac{1}{2}m\pi - \frac{1}{4}\pi\right)$$

Hence:

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial\varphi_7}{\partial r} - ik\varphi_7\right) = 0 \rightarrow$$

$$\sum_{m=0}^{\infty}\sqrt{r}\left(C_1\dot{J}_m(kr) + C_2\dot{Y}_m(kr) - ik\left(C_1J_m(kr) + C_2Y_m(kr)\right)\right)\cosh(kz)\cos(m\theta) = 0$$

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

Thus, the term in the brackets should be equal to zero:

$$(C_2 - iC_1)k\sqrt{\frac{2}{\pi k}}e^{-i\left(kr-\frac{1}{2}m\pi-\frac{1}{4}\pi\right)} = 0 \rightarrow C_2 = iC_1$$

Hence:

$$\varphi_7(r,\theta,\mathrm{z}) = \sum_{m=0}^{\infty}(C_1J_m(kr) + iC_1Y_m(kr))\cosh(kz)\cos(m\theta) \rightarrow$$

$$\boxed{\varphi_7(r,\theta,\mathrm{z}) = \sum_{m=0}^{\infty} C_1H_m(kr)\cosh(kz)\cos(m\theta)}$$

It was assumed that k real number. For the solution to be complete we should investigate whether there are Laplace solutions that satisfy the boundary conditions for which k imaginary number, $k=i\alpha$

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

Equation (6) can be rewritten as:

$$\frac{\partial^2 \mathrm{R}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathrm{R}}{\partial r} - (\alpha^2 + \frac{m^2}{r^2})\mathrm{R} = 0 \qquad (7)$$

We derive from Equation (7) that:

$$\mathrm{R}(r) = (C_1 I_m(\alpha_n r) + C_2 K_m(\alpha_n r)) \cos(\alpha_n z) \cos(m\theta) \qquad (8)$$

Where α_n are infinite real roots of equation: $\omega^2 + \alpha g tan(ad) = 0$, which are formed by:

$$\alpha_n = \frac{n\pi}{d} - \frac{\omega^2}{g}\frac{1}{n\pi}, n = 1,2,3 \ldots$$

For the boundary condition to hold at infinity and the potential function to remain closed, then $C_1 = 0$, since $I_m(\alpha_n r)|_{r\to\infty} \to \infty$, whereas $K_m(\alpha_n r)|_{r\to\infty} \to 0$, Hence:

$$\varphi_7(r, \theta, \mathrm{z}) = \sum_{m=0}^{\infty} \left(A_m H_m(kr) \cosh(kz) + \sum_{n=1}^{\infty} B_{mn} K_m(\alpha_n r) \cos(\alpha_n z) \right) \cos(m\theta)$$

Vertical Bottom Seated Cylinder (McCamy-Fuchs)

Intoducing $\alpha_0 = -ik$, and considering that:

$\tan(z) = -itanh(iz),\ \sin(z) = -isinh(iz), \cos(z) = \cosh(z)$

From equation: $\omega^2 + \alpha_n gtan(ad) = 0$ we can derive the dispersion equation.

Therefore the velocity potential can be written as:

$$\boxed{\varphi_7(r,\theta,\mathrm{z}) = \sum_{m=0}^{\infty}\left(\sum_{n=0}^{\infty} B_{mn}\, K_m(\alpha_n r)\cos(\alpha_n z)\right)\cos(m\theta)}$$

It remains to determine B_{mn} which results from the boundary condition on the wetted surface of the body.

$$\frac{\partial\varphi_7}{\partial r} = -\frac{\partial\varphi_0}{\partial r}, r = a$$

$$\sum_{n=0}^{\infty} \alpha_n B_{mn}\, K'_m(\alpha_n r)\cos(\alpha_n z) = \frac{gH}{2\omega}\varepsilon_m i^{m+1} k J'_m(kr) \qquad (9)$$

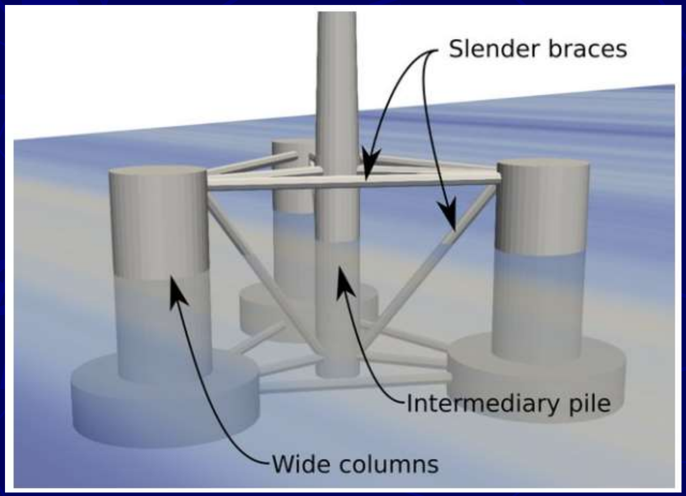
Vertical Bottom Seated Cylinder (McCamy-Fuchs)

From Eq.(7) we can evaluate the form of the velocity potential:

$$\varphi = \varphi_0 + \varphi_7 = -i\frac{g}{\omega}\frac{H}{2}\frac{\cos h(kz)}{\cos h(kd)}\sum_{m=0}^{\infty}\varepsilon_m i^m\left(J_m(kr) - \frac{J'_m(ka)}{H'_m(ka)}H_m(kr)\right)\cos(m\theta)$$

The case we examined is the simplest, since the cylinder is considered compact and the solution was only for the diffraction problem!

The solution process becomes very demanding when we need to consider more complex geometry body/bodies.



Vertical Cylinder – McCamy & Fuchs

The exciting forces $F_x(t) = Re[f_x e^{-i\omega t}]$ can be written as:

$$f_x = -\iint_S Pn_x dS = -i\omega\rho a \int_{z=0}^{z=d} \int_{\theta=0}^{\theta=2\pi} \varphi cos\theta d\theta dz \rightarrow \frac{f_x}{\pi\rho g a^2 \left(\frac{H}{2}\right)} = \frac{4}{\pi(ka)^2 H_1'(ka)} \tanh(kd)$$

The exciting moments $M_y(t) = Re[m_y e^{-i\omega t}]$ can be written as:

$$m_y = -\iint_S Pzn_x dS = -i\omega\rho a \int_{z=0}^{z=d} \int_{\theta=0}^{\theta=2\pi} \varphi z cos\theta d\theta dz \rightarrow \frac{m_y}{\pi\rho g a^3 \left(\frac{H}{2}\right)}$$

$$= \frac{4}{\pi(ka)^3 H_1'(ka)} \left(kd \tanh(kd) - 1 + \frac{1}{\cosh(kd)} \right)$$

The horizontal exciting forces per unit length equal to:

$$\frac{dF_x(t)}{dz} = \pi\rho a^2 \frac{4}{\pi(ka)^2 [(J_1'(ka))^2 + (Y_1'(ka))^2]^{\frac{1}{2}}} \omega^2 \left(\frac{H}{2}\right) \frac{\cosh(kz)}{\sinh(kd)} \sin(\omega t - \varepsilon)$$

Vertical Cylinder – McCamy & Fuchs

The previous relation can be written as:

$$\frac{dF_x(t)}{dz} = \pi\rho a^2 \frac{4}{\pi(ka)^2} A(ka)\omega^2 \left(\frac{H}{2}\right) \frac{\cosh(kz)}{\sinh(kd)} \sin(\omega t - \varepsilon)$$

where $A(ka) = \left[(J_1'(ka))^2 + (Y_1'(ka))^2\right]^{-\frac{1}{2}}$ $\quad \varepsilon = \arctan\left(\frac{J_1'(ka)}{Y_1'(ka)}\right)$

It is known that:

$$\mathrm{u} = \frac{\partial \Phi}{\partial x} = \frac{H}{2}\omega \frac{\cosh(kz)}{\cosh(kd)} \cos(\omega t), \qquad \frac{\partial \mathrm{u}}{\partial t} = -\frac{H}{2}\omega^2 \frac{\cosh(kz)}{\sinh(kd)} \sin(kx - \omega t)$$

$$\sin(\omega t - \varepsilon) = \sin(\omega t)\cos(\varepsilon) - \cos(\omega t)\sin(\varepsilon)$$

Substituting in the above Eq.

$$\frac{dF_x(t)}{dz} = \pi\rho a^2 C_m \frac{\partial \mathrm{u}}{\partial t} + \mathrm{b}\pi\rho a^2 \omega \mathrm{u} \qquad C_m = C_{m1} cos\varepsilon = \frac{4}{\pi(ka)^2} \frac{Y_1'(ka)}{(J_1'(ka))^2 + (Y_1'(ka))^2},$$

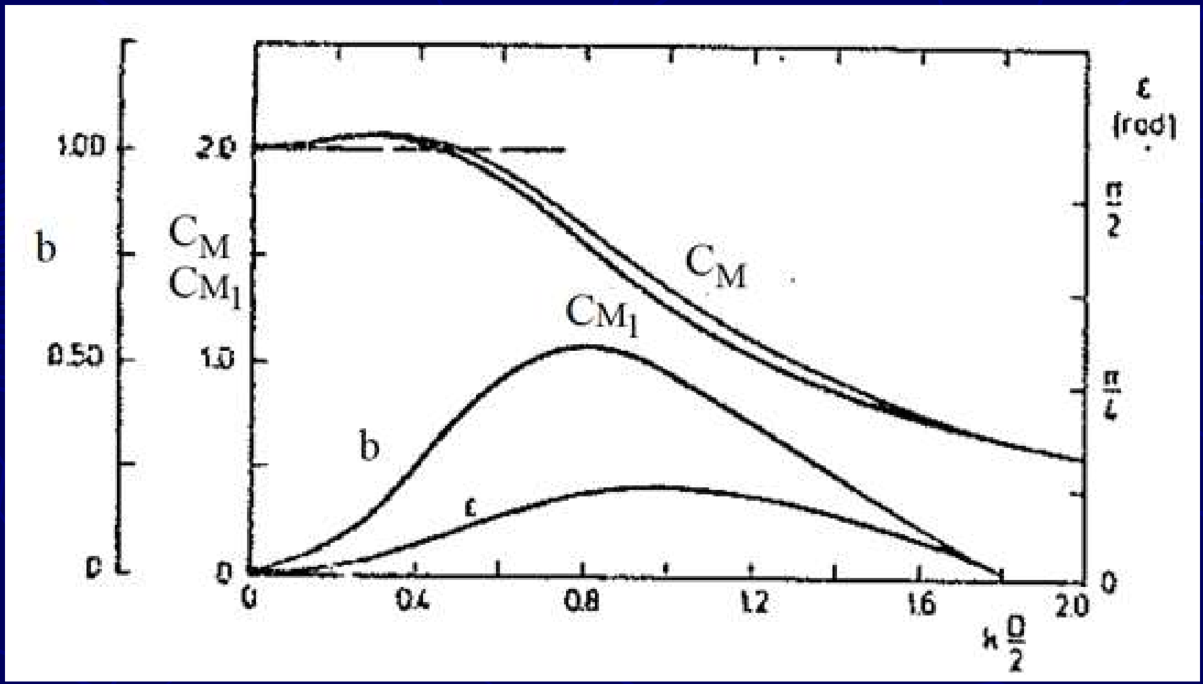
$$\mathrm{b} = C_{m1} sin\varepsilon = \frac{4}{\pi(ka)^2} \frac{J_1'(ka)}{(J_1'(ka))^2 + (Y_1'(ka))^2}$$

Vertical Cylinder – McCamy & Fuchs

In the previous relation: $\frac{dF_x(t)}{dz} = \underbrace{\pi\rho a^2 C_m \frac{\partial u}{\partial t}}_{} + \underbrace{b\pi\rho a^2 \omega u}_{}$

The first part is function of the fluid acceleration (inertia loads)

The second part is function of the fluid velocity (hydrodynamic damping)



Dual Chamber – OWC Converter

The fluid flow around the OWC can be described by the potential function:

$$\varphi = \varphi_0 + \varphi_7 + p_{in0}\varphi_P + \sum_{j=1}^{5} \dot{x}_{j0}\varphi_j = \varphi_D + p_{in0}\varphi_P + \sum_{j=1}^{5} \dot{x}_{j0}\varphi_j$$

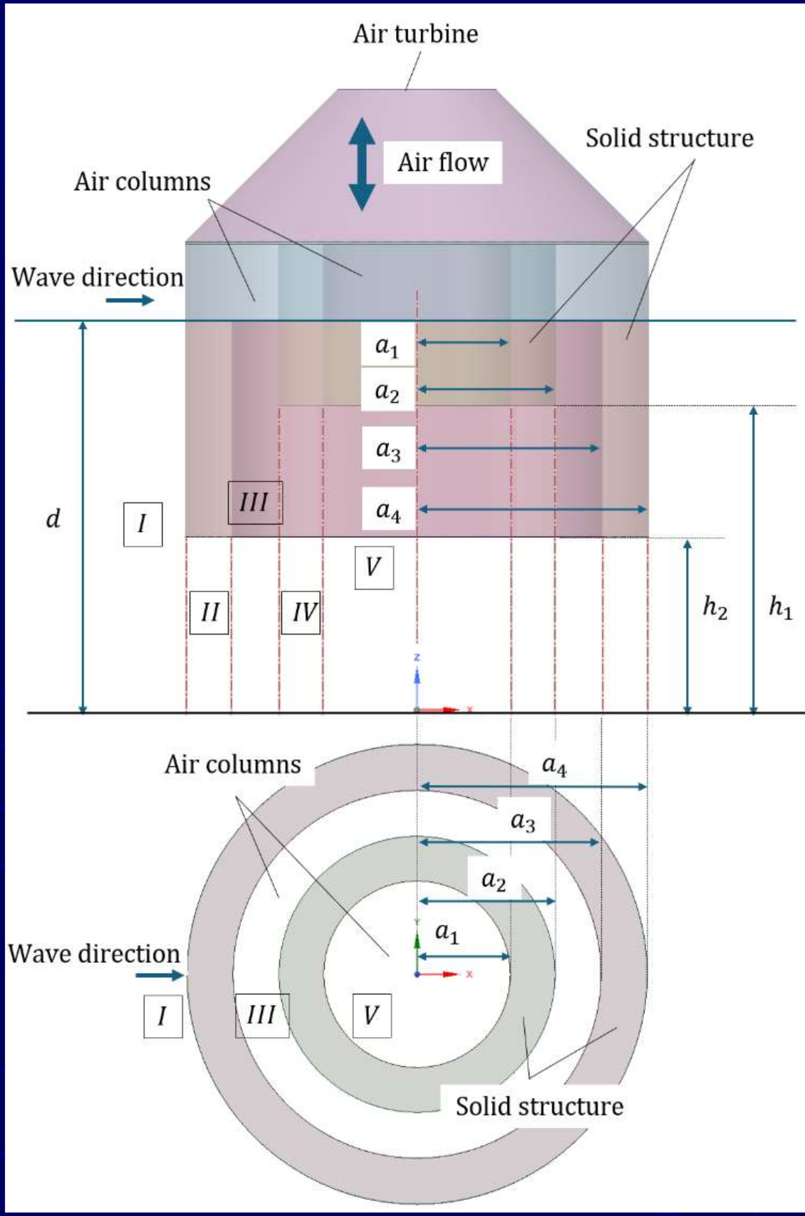
The potentials φ_j (*j=0,* 7, 1,…,5, *P*) have to satisfy the:

- Laplace equation;
- The kinematic boundary conditions on the sea bottom and on the mean body's wetted surface;
- The linearized boundary condition at the outer and inner free sea surface:

$$\omega^2\varphi_j - g\frac{\partial\varphi_j}{\partial z} = 0, j\text{=}0,\ 1,\ 3,\ 5,\ 7, \text{ for } r \geq a_4;\ 0 \leq r \leq \alpha_1$$

$$\omega^2\varphi_p - g\frac{\partial\varphi_p}{\partial z} = 0, \text{ for } r \geq a_4$$

$$\omega^2\varphi_p - g\frac{\partial\varphi_p}{\partial z} = -\frac{i\omega}{\rho}, \text{ for } 0 \leq r \leq \alpha_1, \alpha_2 \leq r \leq \alpha_3$$



Dual Chamber – OWC Converter

The fluid flow around the OWC can be described by the potential function:

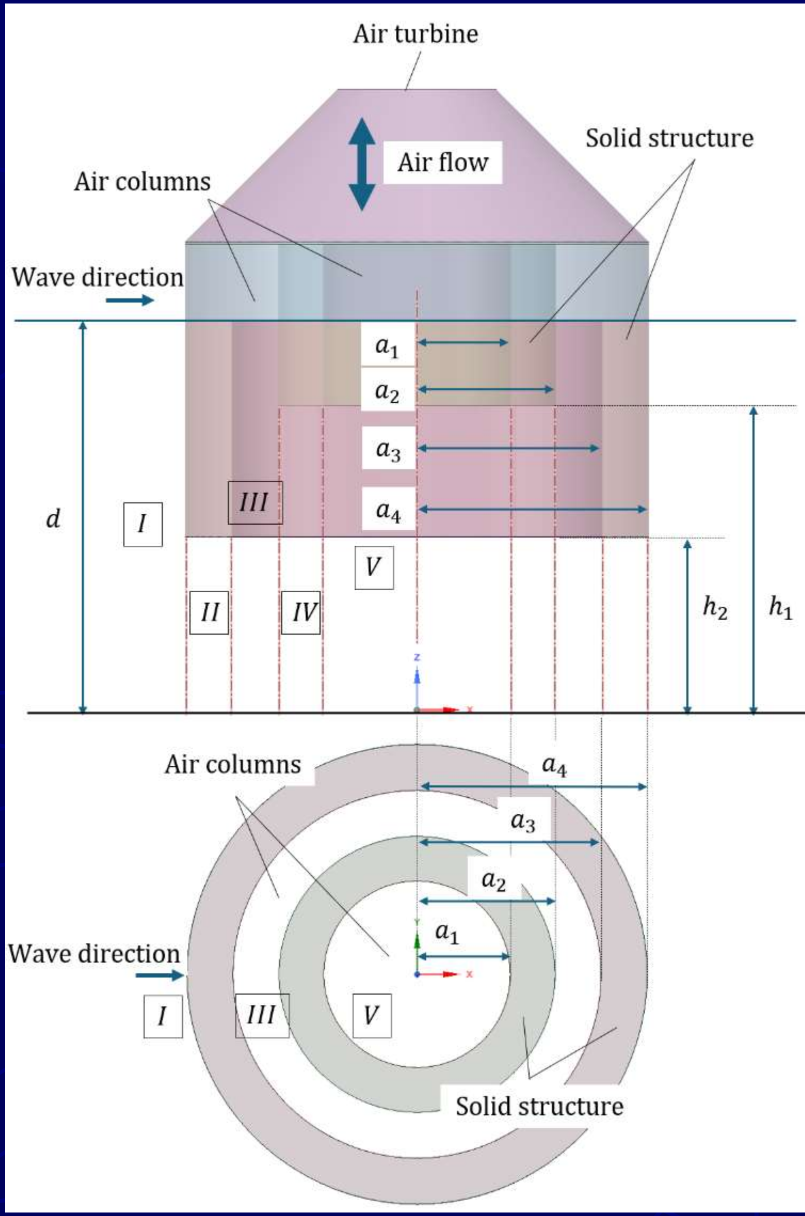
$$\varphi = \varphi_0 + \varphi_7 + p_{in0}\varphi_P + \sum_{j=1}^{5} \dot{x}_{j0}\varphi_j = \varphi_D + p_{in0}\varphi_P + \sum_{j=1}^{5} \dot{x}_{j0}\varphi_j$$

The potentials φ_j (*j=0,* 7, 1,…,5, *P*) have to satisfy the:

- Laplace equation;
- The kinematic boundary conditions on the sea bottom and on the mean body's wetted surface;
- The linearized boundary condition at the outer and inner free sea surface:

$$\omega^2\varphi_j - g\frac{\partial \varphi_j}{\partial z} = 0, j\text{=0, 7, for } r \geq a_4$$

$$\omega^2\varphi_j - g\frac{\partial \varphi_j}{\partial z} = 0,\ j\text{=1,3,5; for } r \geq a_4$$



Dual Chamber – OWC Converter

The diffraction and the radiation potentials in each fluid region ℓ (ℓ=*I, II, III, IV*) can be expressed as follows:

$$\varphi_D(r,\theta,z) = -i\omega\frac{H}{2}\sum_{m=0}^{\infty} i^m\,\Psi^{\ell}_{D,m}(r,z)\cos(m\theta)\,, \qquad \varphi_j(r,\theta,z) = \Psi^{\ell}_{j,m}(r,z)\cos(m\theta)$$

$$\varphi_P(r,\theta,z) = \frac{1}{i\omega\rho}\Psi^{\ell}_{P,0}(r,z)\cos(m\theta)$$

The velocity potentials $\varphi_k, k = D, 1{,}3{,}5, P$ and their derivatives must be continuous at the vertical boundaries of neighboring fluid regions, i.e.:

$$\Psi^{II}_{k,m}(a_4,z) = \Psi^{I}_{k,m}(a_4,z)$$

$$\Psi^{II}_{k,m}(a_3,z) = \Psi^{III}_{k,m}(a_3,z)$$

$$\Psi^{III}_{k,m}(a_2,z) = \Psi^{IV}_{k,m}(a_2,z)$$

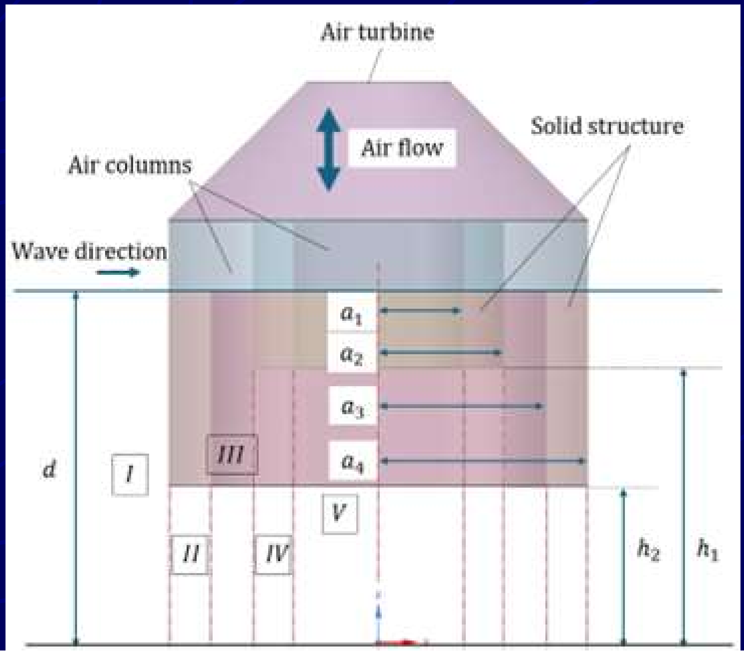
$$\Psi^{V}_{k,m}(a_1,z) = \Psi^{IV}_{k,m}(a_1,z)$$

$$\left.\frac{\partial\Psi^{II}_{k,m}}{\partial r}\right|_{r=a_4} = \left.\frac{\partial\Psi^{I}_{k,m}}{\partial r}\right|_{r=a_4}$$

$$\left.\frac{\partial\Psi^{II}_{k,m}}{\partial r}\right|_{r=a_3} = \left.\frac{\partial\Psi^{III}_{k,m}}{\partial r}\right|_{r=a_3}$$

$$\left.\frac{\partial\Psi^{III}_{k,m}}{\partial r}\right|_{r=a_2} = \left.\frac{\partial\Psi^{IV}_{k,m}}{\partial r}\right|_{r=a_2}$$

$$\left.\frac{\partial\Psi^{V}_{k,m}}{\partial r}\right|_{r=a_1} = \left.\frac{\partial\Psi^{IV}_{k,m}}{\partial r}\right|_{r=a_1}$$



Dual Chamber – OWC Converter

The air volume flow produced by the oscillating internal water surface in the OWC device, denoted by:

$$q = \iint_S u_z \, dS_q = \iint_S \frac{\partial \varphi}{\partial z} r dr d\theta$$

can be decomposed, into three terms associated with the diffraction, q_D, the motion-dependent, q_R and the pressure-dependent, q_p, radiation problems:

$$q = q_D + \dot{x}_{30} q_3 - \dot{x}_{30} S_i + p_{in0} q_P$$

Assuming that a Wells type air turbine is placed in a duct between the chamber and the outer atmosphere, of the device, and it is represented by a complex pneumatic admittance, Λ, then the total volume flow *q* equals to :

$$q = \Lambda \cdot p_{in}$$

For $\Lambda_{opt} = \left| q_p \right|$ the absorbed power by the OWC maximizes.

Dual Chamber – OWC Converter

The exciting forces on the device F_i are calculated as:

$$F_i^D = f_i^D \cdot e^{-i\omega} = \iint_S i\omega\rho\,\varphi_D \cdot e^{-i\omega t} \cdot ndS$$

$$M_i^D = m_i^D \cdot e^{-i\omega t} = \iint_S i\omega\rho\,\varphi_D \cdot e^{-i\omega t} \cdot (nxr)dS$$

The hydrodynamic reaction forces and moments $F_{i,j}$, *i,j*= 1,3,5; of the device in the *i*-th direction due to the unit forced oscillation of the device on the *j*-th direction can be obtained as:

$$F_{i,j}^R = -i\omega\rho\dot{x}_{j0} \iint_S \varphi_j \cdot e^{-i\omega t} n_i dS = \omega^2 \left(a_{i,j}^R + \frac{1}{\omega} b_{i,j}^R \right) \cdot x_{j0} \cdot e^{-i\omega t}, \qquad i,j = 1,3,5$$

The pressure hydrodynamic reaction forces and moments, F_3^p of the OWC device in the heave direction due to its inner air pressure can be obtained as:

$$F_3^P = -i\omega\rho p_{in0} \iint_S \varphi_P \cdot e^{-i\omega t} \cdot n_3 dS = (e_3^P + id_3^P) \cdot p_{in0} \cdot e^{-i\omega t}$$

Dual Chamber – OWC Converter

The equations of motion that govern the linear dynamic motions of the OWC device are summarized as:

$$\sum_{j=1}^{5}(m + a_{k,j}^{R})\ddot{x}_{j0} + \left(b_{k,j}^{R} + b_{k,j}^{m}\right)\dot{x}_{j0} + (c_{k,j} + c_{kj}^{m})x_{j0} + \delta_{k,3}(e_3^p - id_3^p)p_{in0} = f_k + \delta_{k,3}f_{MP}$$

Whereas the volume flow equation can be written as:

$$\Lambda \cdot p_{in0} = q_D + q_R + p_{in0} \cdot q_P = q_D + q_3 \cdot \dot{x}_{30} - \dot{x}_{30}S_i + p_{in0} \cdot q_P$$

The time – averaged power absorbed by the OWC from the waves can be obtained as:

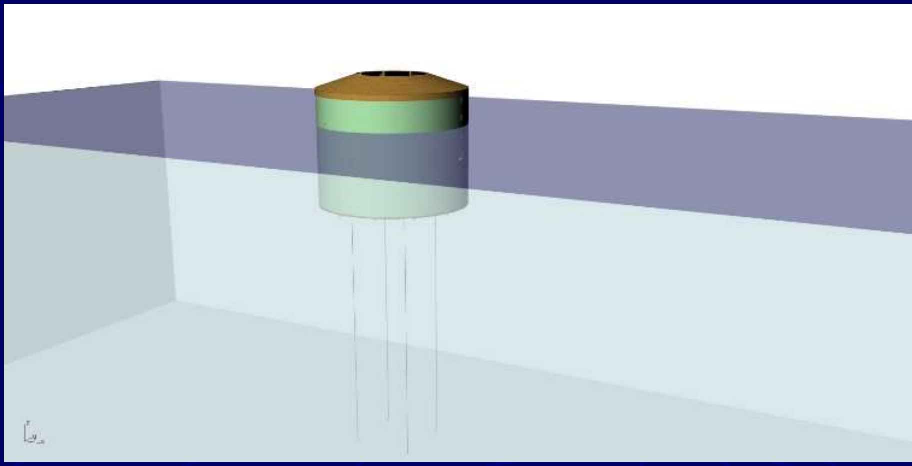
$$P_{out} = \frac{1}{2}Re[\Lambda \cdot |p_{in0}|^2]$$

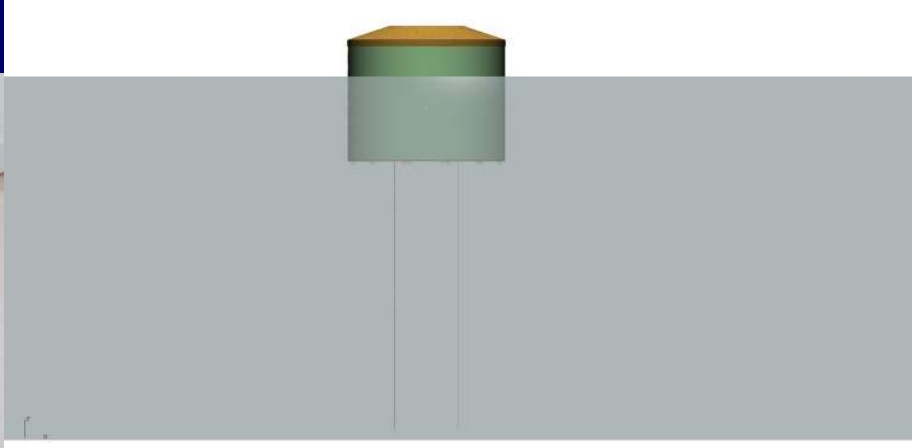
Here: $\Lambda = \frac{KD}{N\rho_{air}^{0}} - i\omega\frac{V_0}{c_{air}^{2}\rho_{air}^{0}}$, where:

- N : the rotational speed of turbine blades
- D : the outer diameter of turbine rotor
- V_0 : the device's air chamber volume
- c_{air} : the sound velocity in air

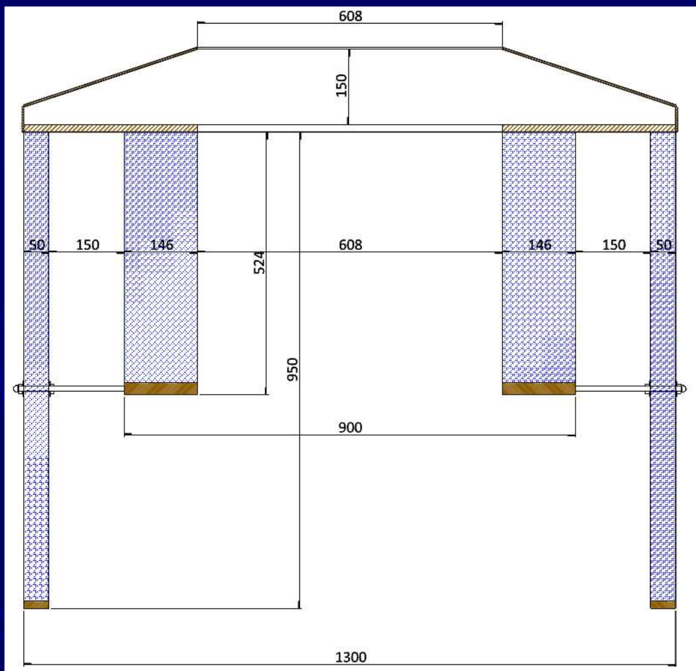
Dual Chamber – OWC Converter

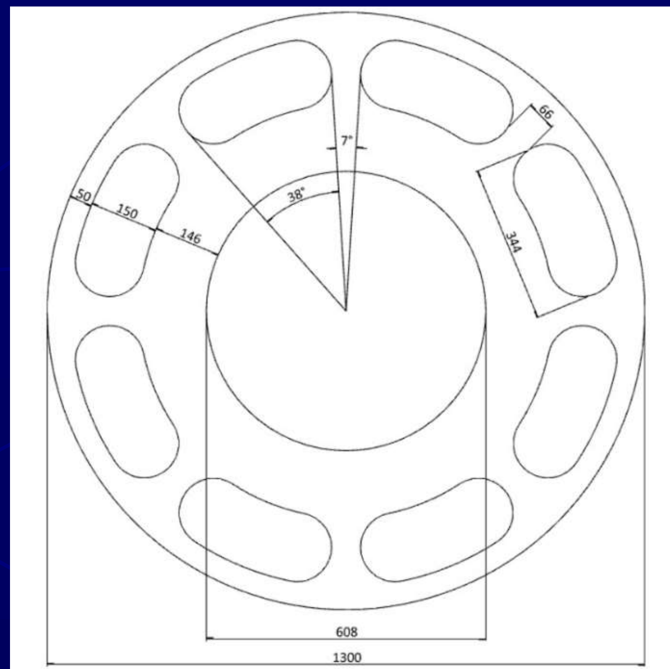
Dimensions	Full Scale	1:20
Depth of the outer pontoon below Sea Water Level (SWL)	14m	0.7m
Depth of the inner pontoon below SWL	5.5m	0.275
Elevation of the oscillating chamber above SWL	5m	0.25m
Outer radius of outer pontoon	13m	0.65m
Inner radius of outer pontoon	12m	0.6m
Outer radius of inner pontoon	9m	0.45m
Inner radius of inner pontoon	6.08m	0.304m
Height of the conical dome	3m	0.15m
Initial diameter of the orifice	6.08m	0.304m

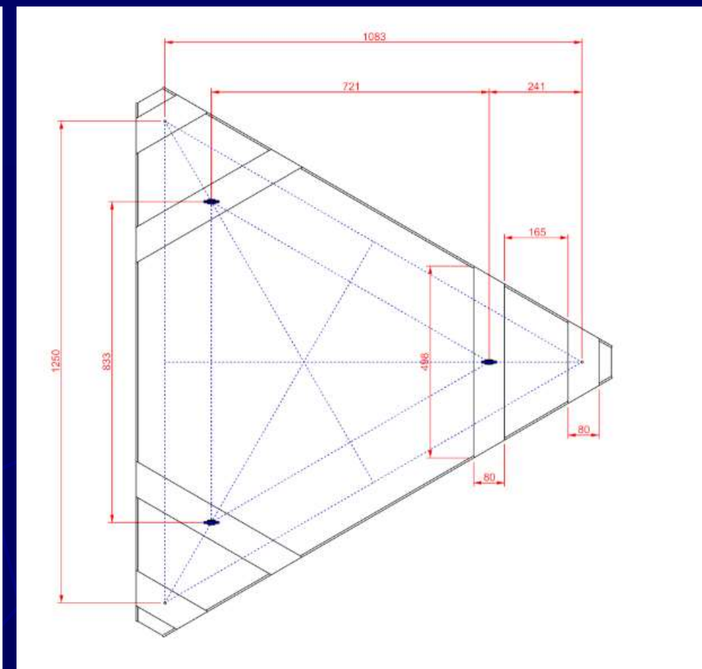


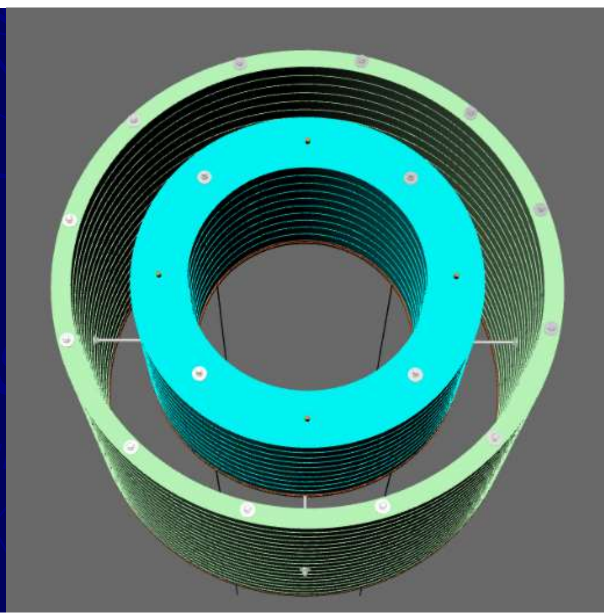


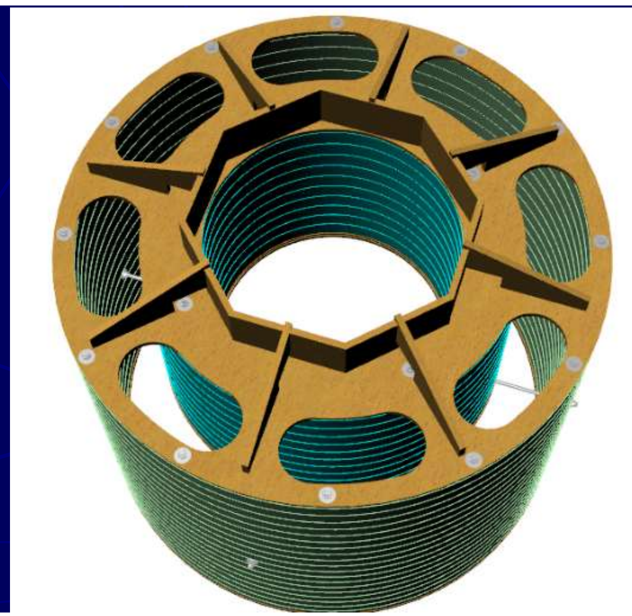
Dual Chamber – OWC Converter

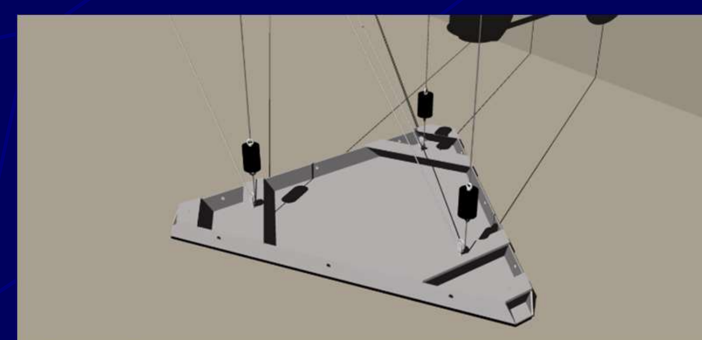






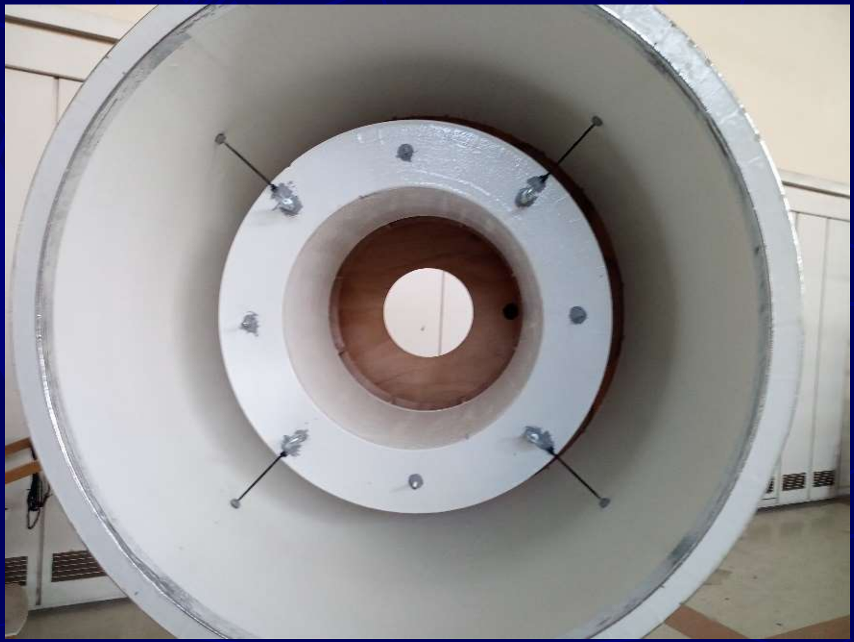


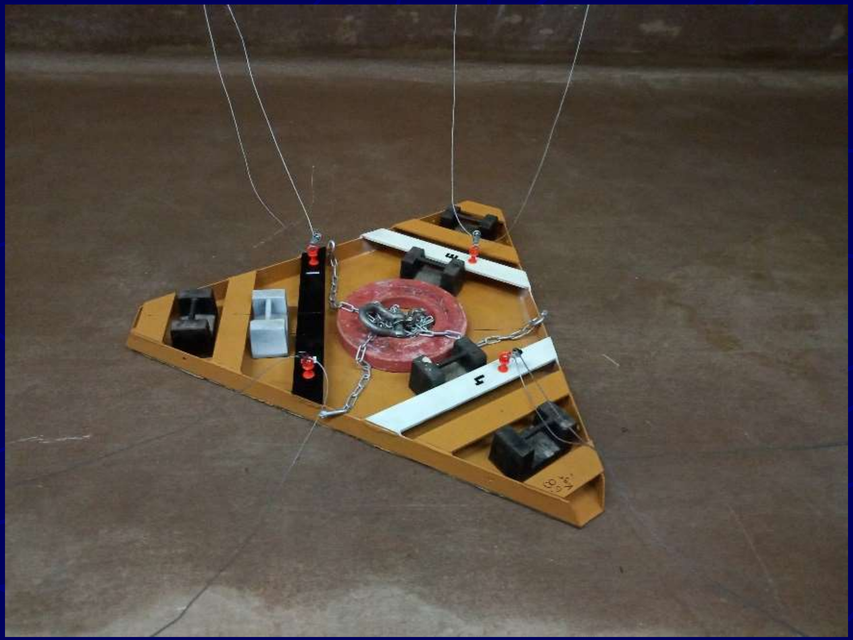




Dual Chamber – OWC Converter

Dual Chamber – OWC Converter





Dual Chamber – OWC Converter

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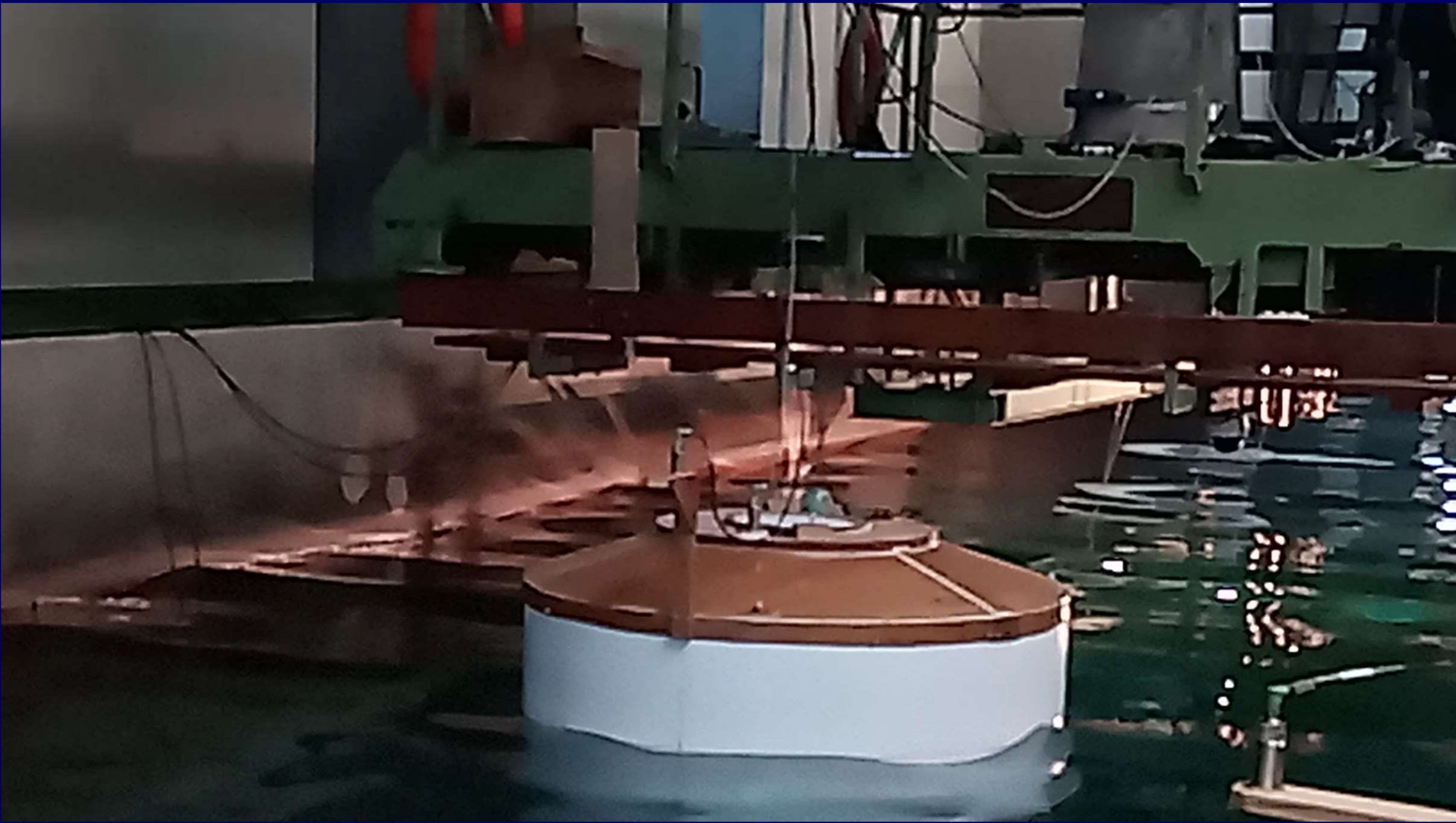
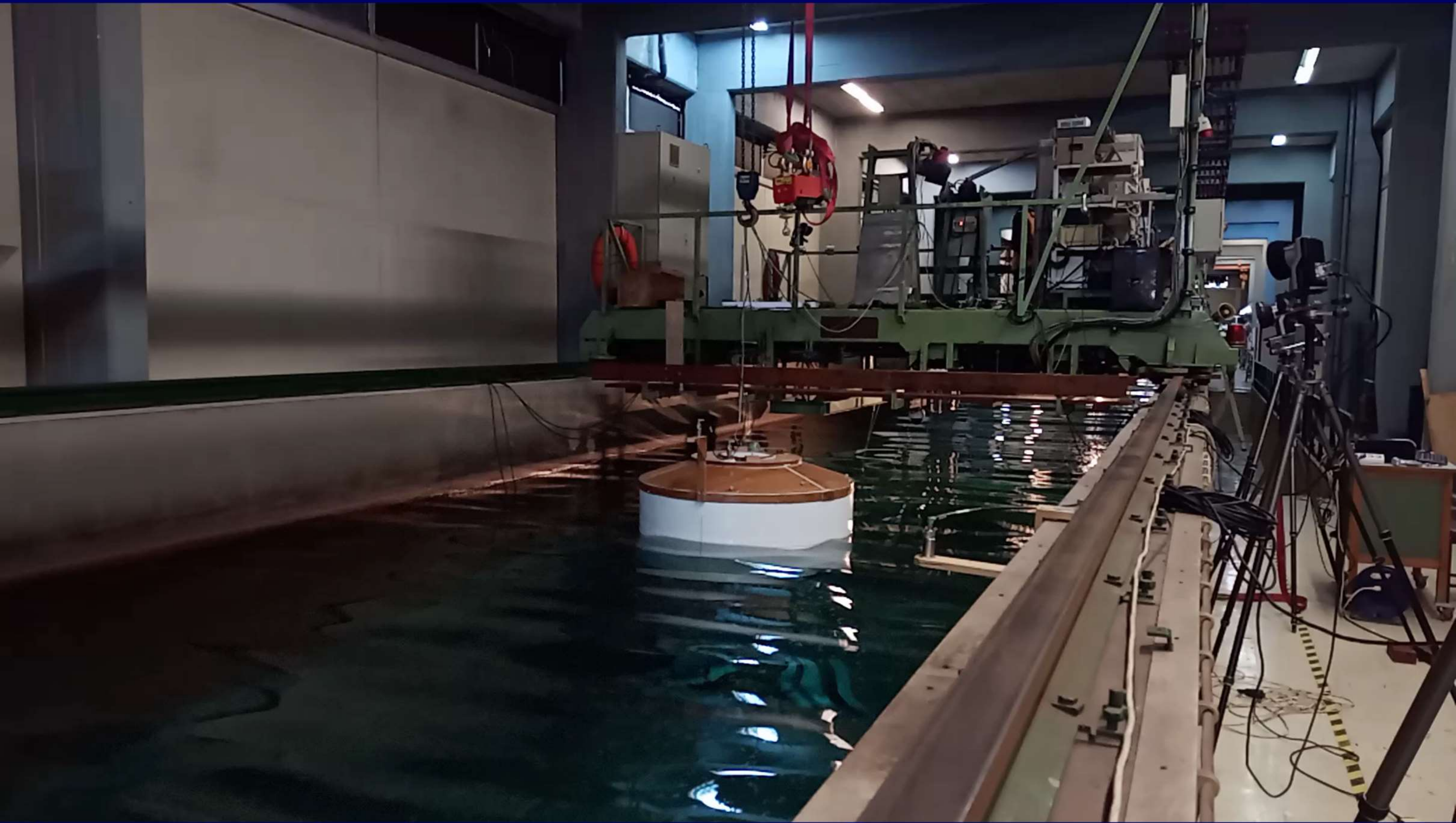


Dual Chamber – OWC Converter



Dual Chamber – OWC Converter

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